Effect of Ion-Neutral Collisions on Plasma Rotation

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By calculating the BOLTZMANN collision integral for collisions between ions and neutral particles, two different formulae for the rotation velocity v_Θ of a cylindrical plasma column driven by $I_r B_z$-forces have been derived. For $v_\Theta < v_{\text{th}}$ the rotation velocity is proportional to $I_r B_z$, whereas for $v_\Theta > v_{\text{th}}$ one gets a dependency $v_\Theta \sim (I_r B_z)^{1/2}$. v_{th} denotes the mean thermal velocity of the ions.

In experiments with a rotating plasma driven through a neutral gas by the $I_r B_z$-force, different variations of the rotation velocity v_Θ with the magnetic field strength B_z have been observed. Probe measurements made in a PENNING discharge [1] at Fontenay-aux-Roses for a limited range of magnetic fields (1000 – 3000 gauss) indicated the rotation frequency to be roughly proportional to the square root of the magnetic field strength. In a similar device [2] the plasma rotation velocity was measured spectroscopically from the DOPPLER shift of spectral lines. The rotation velocity was found to be proportional to $(B/r)^{1/2}$, r denoting the radial distance of the observed plasma shell.

A larger range of magnetic fields including field strengths down to 50 gauss was investigated by CHEN [3] with probe diagnostics in the "Short PIG" helium discharge [4]. For weak fields the rotation frequency observed in this experiment was directly proportional to the field strength B_z, while for strong fields the observed dependency was similar to that reported in ref. [1].

In both probe experiments (ref. [1] and [3]) the signal is supposed to be caused by an azimuthal "flute like" anisotropy [5] of the plasma column rotating roughly with its macroscopic velocity.

We propose an explanation of both the $B_z^{1/2}$- and the B_z-dependency of the rotation velocity by simply calculating the collision integral in the stationary momentum equation, assuming that the dominant cross sections are those of charge exchange and elastic collisions between ions and neutrals. We further assume [2] that the mean free path of the neutral particles is large compared with the diameter of the plasma column. Thus, there is a strong coupling between the neutral particles and the walls, keeping the neutral gas at rest and approximately at the wall temperature. The wall temperature is negligibly small compared with the temperature of the charged particles. A charge exchange collision is then equivalent to a complete loss of momentum of the ion. Since the fast neutral particles escape rapidly from the plasma region after a collision, no inverse encounters (between fast neutrals and slow ions) need to be taken into account.

The collision integral is now calculated assuming that the charge exchange cross section is a constant in the velocity range of interest and that the elastic collisions can be treated in the rigid sphere approximation [6].

A rigorous calculation of the collision integral would require the knowledge of the distribution functions which should be determined as stationary solutions of kinetic equations of the BOLTZMANN type. However, it is sufficient for our purposes to postulate the ionic distribution to be locally maxwellian around the rotation velocity v_Θ and the neutral particles to be completely at rest.

The azimuthal component of the momentum equation is in our case

$$i_r B_z = \int (m_i v)_\Theta \left[\left(\frac{\partial f_i}{\partial t}\right)_{\text{exch}} + \left(\frac{\partial f}{\partial t}\right)_{\text{elast}} \right] d^3 v \qquad (1)$$

where i_r denotes the radial current density, B_z the the azimuthal component of the momentum variable and longitudinal magnetic field, m_i the ion mass, $(m_i v)_\Theta$ f_i the ion distribution function.

Momentum transfer by collisions between electrons and neutral atoms is neglected in this equation. This approximation is valid if the electron temperature does not greatly exceed the ion temperature, so that the contribution of the electrons to the mean momentum is then small compared with that of the ions.

The BOLTZMANN collision integral yields for charge exchange and elastic collisions respectively

$$\left(\frac{\partial f_i}{\partial t}\right)_{\text{exch}} = f_0 \int f_i \, |\boldsymbol{v}| \, \sigma_{\text{exch}} \, d^3 v \,, \qquad (2)$$

$$\frac{\partial f_i}{\partial t}_{\text{elast}} = \frac{\sigma_{\text{elast}}}{2\pi} \iint |(\boldsymbol{v}-\boldsymbol{w})\cdot\boldsymbol{a}| \left\{ f_i \left(\boldsymbol{v}+\boldsymbol{a}\cdot(\boldsymbol{w}-\boldsymbol{v})\boldsymbol{a}\right) \cdot f_0 \left(\boldsymbol{w}-\boldsymbol{a}\cdot(\boldsymbol{w}-\boldsymbol{v})\boldsymbol{a}\right) - f_i(\boldsymbol{v}) \, f_0(\boldsymbol{w}) \right\} d^2 a \, d^3 \omega \,. \qquad (3)$$

($\boldsymbol{a}$ denoting a unit vector, the integration with respect to $\boldsymbol{a}$ is carried out over a sphere).

In these expressions the distribution functions f_i and f_0 of the ions and the neutral particles have to be expressed as follows

$$f_i(\boldsymbol{v}) = n_i (\beta/\pi)^{3/2} \exp\{-\beta(\boldsymbol{v}-\boldsymbol{v}_\Theta)^2\} \,, \qquad (4\text{ a})$$

$$f_0(\boldsymbol{v}) = n_0 \, \delta(\boldsymbol{v}) \qquad (4\text{ b})$$

where $\beta = m_i/(2 k T_i)$, and n_i, n_0 are the number den-

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sities of the ions and the neutral particles respectively.

Carrying out the integrations in equs. (1), (2) and (3) one gets

$$i_r B_z = \frac{1}{\beta \sqrt{\pi}} \sigma n_i n_0 m_i F(v_\Theta \sqrt{\beta}) \tag{5}$$

where

$$\sigma = \sigma_{exch} + \frac{1}{2} \sigma_{elast} \tag{6}$$

and

$$F(x) = e^{-x^2}\left(x + \frac{1}{2x}\right) + \sqrt{\pi}\,\Phi(x)\left(x^2 + 1 - \frac{1}{4x^2}\right) \tag{7}$$

$$\simeq \begin{cases} \frac{8}{3}(x + \frac{1}{5}x^3) & \text{for} \quad x \ll 1, \\ \sqrt{\pi}(x^2+1) & \text{for} \quad x \gg 1, \end{cases}$$

$$\text{where} \quad \Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt.$$

Integrating equ. (5) in a plasma shell of length L and radial distance r, one gets the following formula for the rotation velocity v_Θ of the sheath:

$$v_\Theta = \left(\frac{2kT_i}{m_i}\right)^{1/2} F^{-1}\left(\frac{I_r B_z}{4\sqrt{\pi}\, r L \sigma n_0 n_i k T_i}\right) \tag{8}$$

where F^{-1} is the *inverse* function of F, as defined by equ. (7). For very low and very high rotation velocities one obtains the formulae

$$v_\Theta \cong \begin{cases} \dfrac{3}{16\sqrt{2\pi}} \cdot \dfrac{I_r B_z}{r L \sigma n_i n_0 \sqrt{m_i k T_i}} & \text{for } v_\Theta \ll \left(\dfrac{2kT_i}{m_i}\right)^{1/2} \quad (9) \\ \left(\dfrac{I_r B_z}{2\pi r L \sigma n_i n_0 m_i} - \dfrac{2kT_i}{m_i}\right)^{1/2} & \text{for } v_\Theta \gg \left(\dfrac{2kT_i}{m_i}\right)^{1/2}. \quad (10) \end{cases}$$

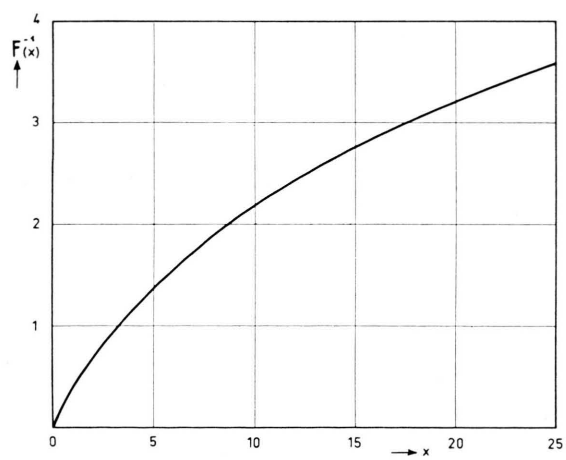

Fig. 1. The function $F^{-1}(x)$.

Here $I_r = 2\pi r L\, i_r(r)$ denotes the *total radial current* which traverses the shell of length L at radial distance r. The function $F^{-1}(x)$ is plotted in Fig. 1, showing the B_z- and the $B_z^{1/2}$-dependency of v_Θ for very small and very strong field strengths respectively, and a mean behaviour between these two ranges.

Note that no z-dependence of v_Θ has been observed in spite of the particular geometry of the electrodes. This may be explained by the large mobility of the charged particles along the magnetic field lines which tends to establish a uniform radial current density along the whole line. Further, there is some evidence for "magnetic rigidity" due to magnetic flux conservation, as has been pointed out in ref. [1]. Following these arguments one would find a rotation of the plasma column with the same frequency for all z even in those cases where the main part of the radial current flows only in the regions near the electrodes.

It should be emphasized that a highly accurate comparison of our formulae with the experimental results [1–3] cannot be carried out since several of the physical quantities in equs. (5), (8), (9) and (10) are not very well known for these experiments. Taking this into consideration the results of this paper are consistent with the experiments.

Formula (10), except the correcting term $2kT_i/m_i$ in the bracket, has already been given in refs. [1, 2] and [7] to explain the experimental results in refs. [1] and [2] for strong magnetic fields. The same good agreement is obtained when comparing the more general formula (8) with the experimental results of ref. [3]: Using the experimental data [3, 4] $I_{total} = 0.4$ amp., $L = 55$ cm, $r = 0.5$ cm, $n_0 = 3 \cdot 10^{14}$ cm^{-3}, $n_i = 10^{12}$ cm^{-3}, kT_e = several eV, there remains still some ambiguity as far as the radial current I_r, the cross section σ and the temperature T_i of the ions are concerned. Taking $\sigma = 3 \cdot 10^{-15}$ cm^2 (ref. [8]), and choosing further $kT_i = 0.10$ eV for example, and $I_r = I_{total}$, the rotation frequencies calculated from equ. (8) agree within 25%, or better, with the measured values (ref. [3]) over the whole range of the magnetic field from 400 gauss to 5000 gauss.

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